Bitcoin and Beyond

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Thanks

I would like to extend a Thank you to:

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I would like to extend a Thank you to:

- Uli Walther – my advisor at Purdue
- Joseph Lipman – Who was kind enough to offer an elliptic curves course.
- The Boston chapter of the National Information Security Group

Outline of topics discussed

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- Elliptic Curves

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- Proof of work and mining
- What's after ECDSA?

ELLIPTIC CURVES ARE BEAUTIFUL!

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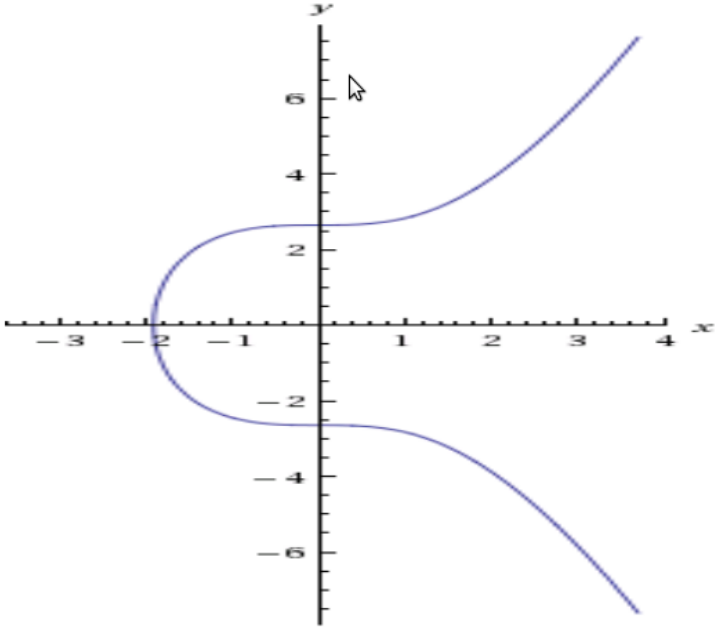
ELLIPTIC CURVES ARE BEAUTIFUL!

$$y^2 = x^3 + 7$$



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We will need a finite arithmetic

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For example addition modulo 5

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$+$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

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It was important that our choice of 5 was prime.

The Elliptic curve used by Bitcoin

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- The prime used is $p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$ *instead of 5 above*
- There is a specific known solution of this curve. We'll discuss this later.

What's the big deal about elliptic curves?

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Why elliptic curves?

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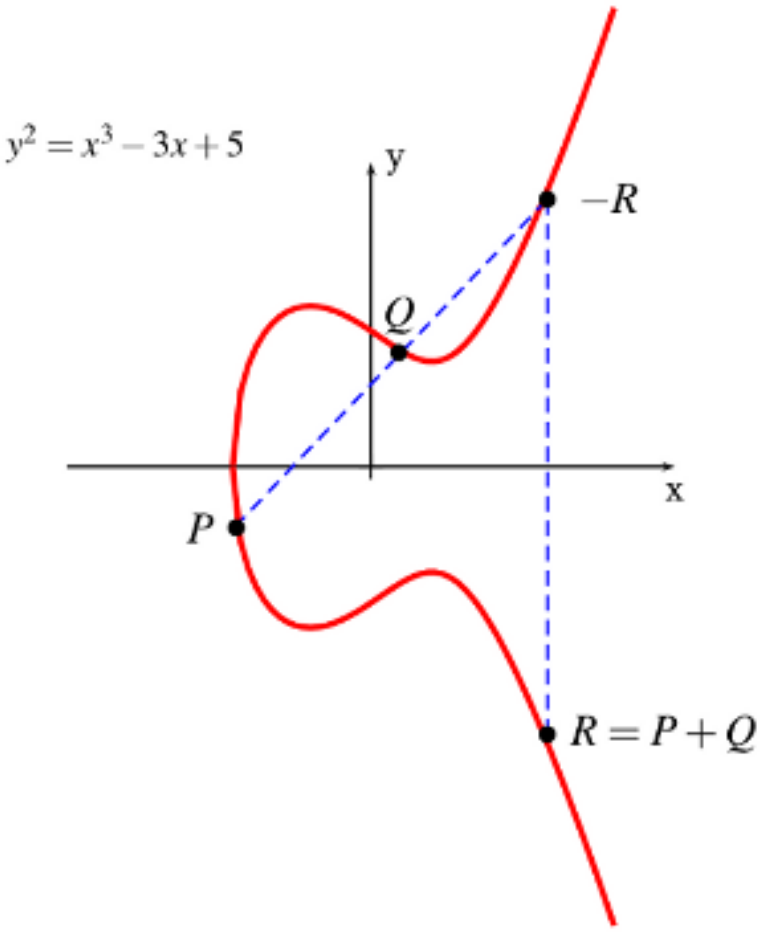
What's the big deal about elliptic curves?

Why elliptic curves?
There is a way to "add" two solutions of an elliptic curve together to get another solution.

Adding points on an Elliptic Curve



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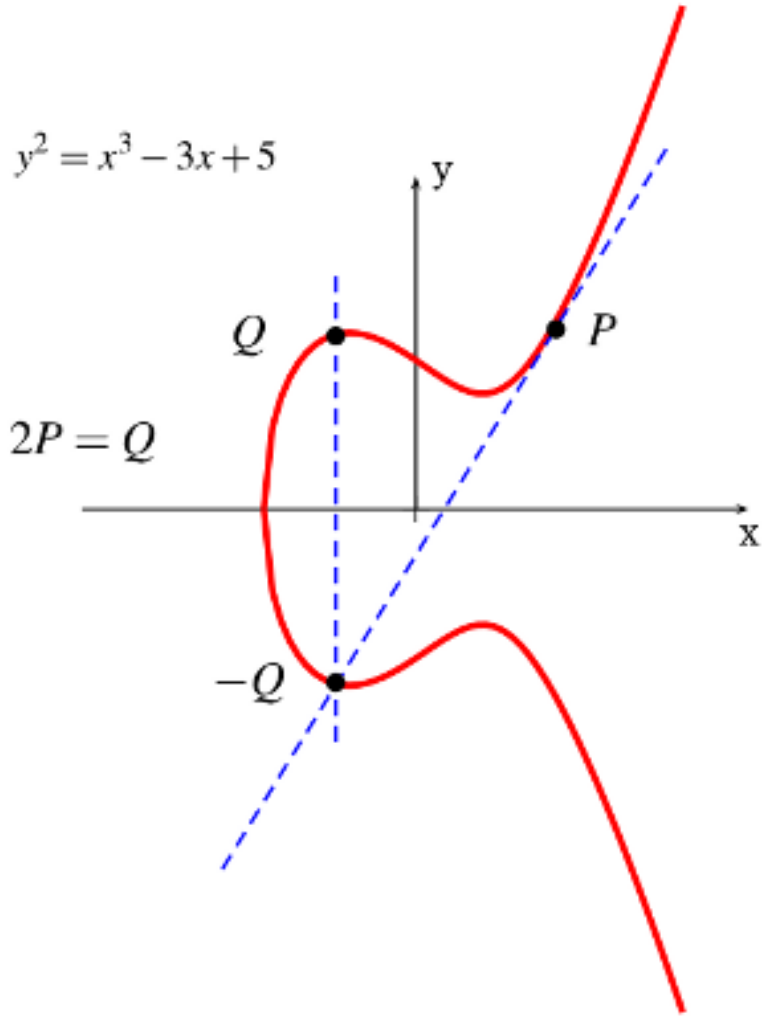

Hash functions: SHA256

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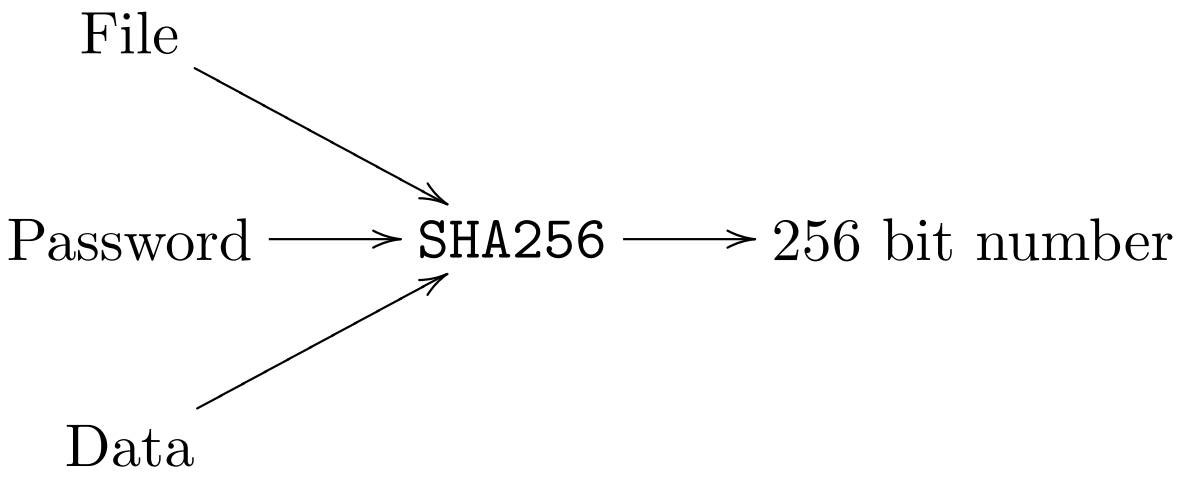

and has a 256 bit output.

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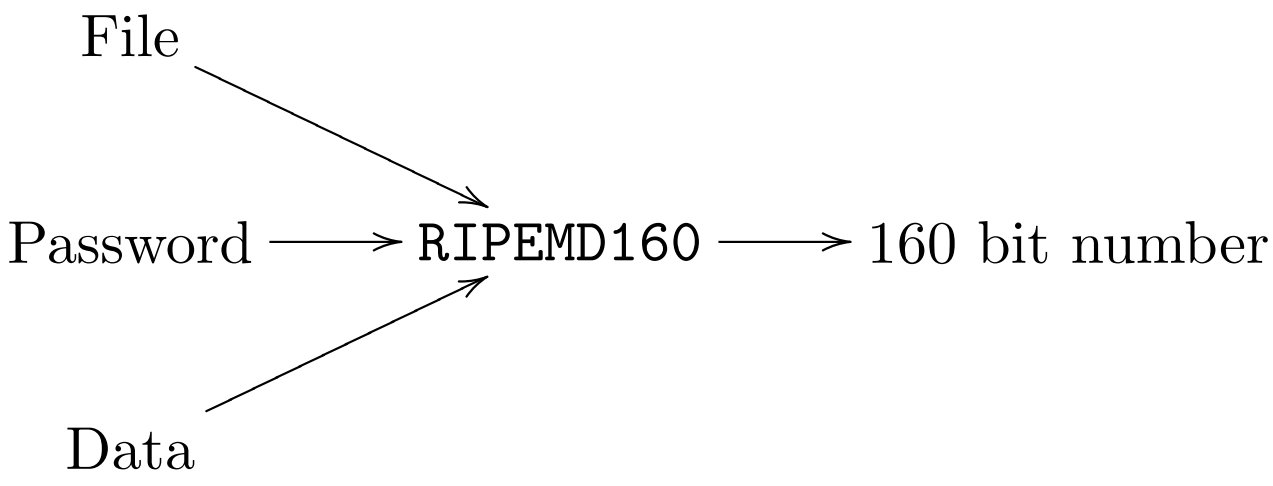
In the case of `SHA256` there are 2^{256} outputs.

Hash functions: RIPEMD160



Public, private keys, and addresses

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- The public key associated to a private key is E added to itself that number of times.
- The address associated to the key is the `RIPEMD160` hash of the `SHA256` hash of the public key.
- An address, `1L5rFEcJUix2b9q6u2n7yqyLEzSxYhB4a1`, is a base 58 encoding of that hash. Always with a 1 at the beginning.

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- Address 1 and then $\texttt{RIPEMD160}(\texttt{SHA256}(P))$ encoded in base 58.

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It is possible to prove you know the private key associated to a public key, without divulging the private key.

Wallets

A bitcoin wallet is:

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Wallets

A bitcoin wallet is:
A file that contains private, public keys and addresses. It usually will contain many keys. It may also contain labeling information.

Digital Signatures

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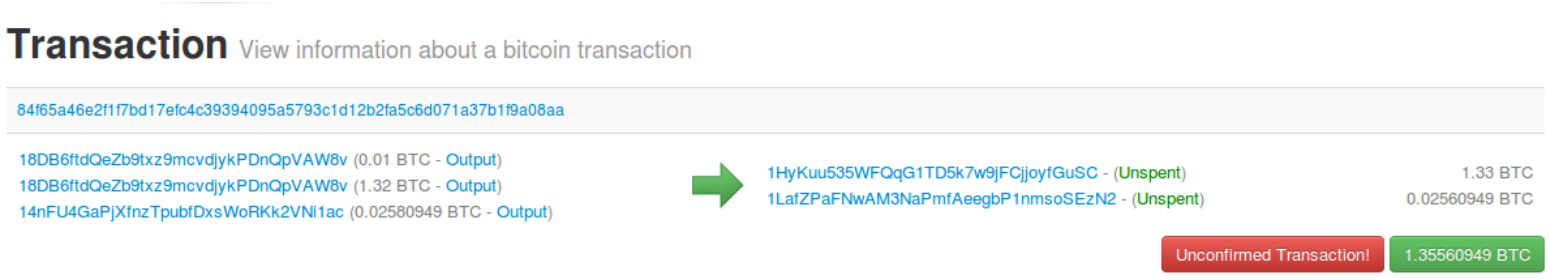
- A transaction can draw from one or more unspent inputs. If it draws from one or more addresses, then multiple signatures are needed.
- A transaction can have one or more outputs.
- An input must be spent in full. Any amount that is too much can be sent back to an address owned by the spender.

A transaction



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- Some transactions that are non-standard and will not be relayed by the network. This helps prevent a DDOS.

Known attacks on ECDSA

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There has been some work done in small characteristic. Since the characteristic that Bitcoin uses is so large $\sim 2^{256}$, this won't work.

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If two signatures are known this method becomes more robust.

For safety sake

Don't use the same address twice.

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Mining

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- Each next block must have the hash of the previous block. It's a chain
- The longest chain is always considered the valid one.

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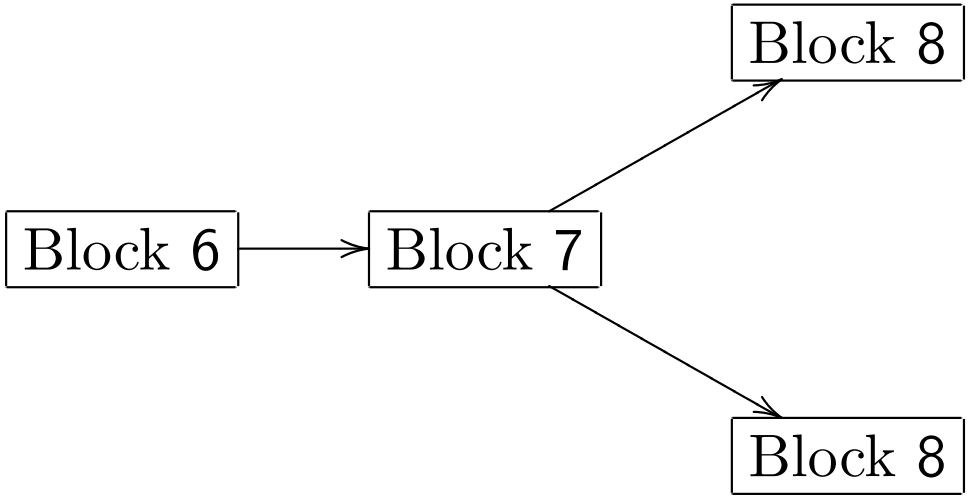

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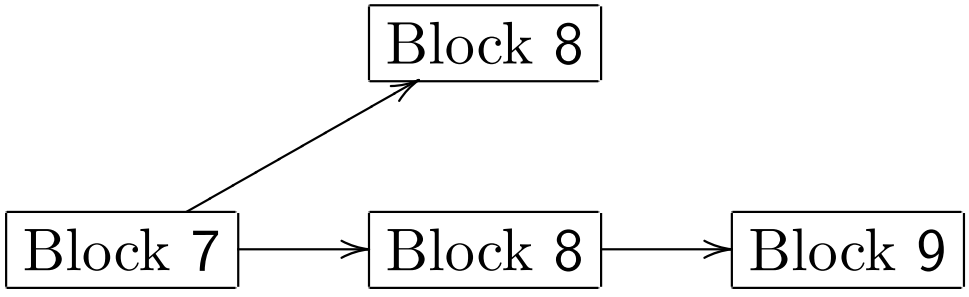

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- Finding a good nonce is called finding a block.

Upgrading ECDSA

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- Lamport public and private keys are too big. Bandwidth will be an issue.
- What if there is a quantum attack on ECDSA. Larger fields won't be much help.
- The current protocol is resistant to an ECDSA attack because of the use of hash functions.